First-order differential equation

Solve the following differential equation:

$$x - \left(x^2 y + y\right) y' = 0$$

Solution

Reorganizing:

$$x\,dx = (x^2y + y)\,dy$$
$$x\,dx = (x^2 + 1)y\,dy$$
$$\frac{x}{x^2+1}\,dx = y\,dy$$

Using substitution to solve the left side:

$$v = x^2 + 1$$
$$dv = 2x\,dx$$
$$\frac{dv}{2} = x\,dx$$

$$\int \frac{1}{v}\,\frac{dv}{2} = 2\ln(v) + C = 2\ln(x^2+1) + C$$

Finally:

$$2\ln(x^2+1) + C = \frac{y^2}{2}$$